ABSTRACT

A mechanistic model was developed from fundamental law of mass diffusion with the aim of describing rate of water loss during osmotic dehydration of red bell pepper at different operating conditions. The developed model was solved analytically (separation of variables technique) and this resulted to Ordinary Differential Equations (ODE). Then, integral properties were adopted to calculate Fourier coefficients of the series. Computer codes were developed in MATLAB for analytical procedure to simulate water loss at various drying operating conditions. The simulated results were compared with experimental data and correlation coefficient for the results gave high values ranging from 0.955 to 0.997. It was observed that simulated results agreed with the experimental. The simulated results could be further used in research and development centre for the design, control and optimization of osmotic dehydration of bell pepper.

KEYWORDS: Osmotic dehydration, simulation, diffusion, optimization and modeling.

1. INTRODUCTION

Red bell pepper is usually scarce in Nigeria during dry season. It is highly perishable with moisture content of about 74% (Oke and Adeniyi, 2013; Ibrahim and Mehmet, 2002). This demand leads to exploring osmotic dehydration (OD) method as pre-drying operation prior to conventional drying methods such as freezing, canning or thermal drying that rely on heating or cooling operations. Osmotic dehydration of bell pepper partially removes free surface water from the tissue. It involves immersion of red bell pepper in a solution (sugar/salt solution) for a given time, with water activities lower than that of the pepper (Le Mague, 1988, Krokida et al., 2000, Park et al., 2002, Siddiq et al., 2012; Oke and Adeniyi, 2013). It gives rise to two major simultaneous counter-current flows: water flows (water loss) from the tissue into the solution and solute from osmotic solution into the pepper, which are both due to the water and solute activity gradients across the interface of the tissue of the pepper and solution. However, rate of water loss during the process is relatively slow; and so, it takes longer time to reach equilibrium water loss.

Rate of water transfer during OD of the pepper depends on various operating conditions: concentration and temperature of osmotic solution, pretreatment of the material prior to osmosis and mass ratio of the solution to bell pepper, process time, agitation, size and shape of bell pepper, type of osmotic agent among others (Lazarides and Mavroudis, 1996, Nieuwenhuijzen et al. 2001,Elez- Martinez et al., 2005, Moazzam, 2012, Oke and Adeniyi, 2013.). Ade-Omowaye et al., 2002 experimentally estimated rate of water loss during the process at different operating conditions. It took much energy with reference to earlier process time to obtain the data, which were insufficient, for design and optimization of the process. However, mathematical modeling could predict water loss from the tissue of red bell pepper at any time and other process conditions without determining water loss at any earlier time.

Accelerated water loss rate during osmotic dehydration of treated carrots with different field strengths was reported by Rastogi et al. (1999). In their work, the effective diffusion coefficient of water was determined using Fickian diffusion model which was reported to increase exponentially with electric field strength. However, it was stated that effective diffusion coefficient was very prominent when field strength greater than 1.09kVcm was applied. Osmotic dehydration model describes water diffusion through the solid into the liquid phase as reported by Lim et al. (2004), Porto and Lisboa (2005) and Rastogi et al., (1999). Fundamental solutions of the diffusion problems for spheres, cylinders flat plates have been provided (Crank, 1992; Gebhart, 1993). Numerical and analytical solutions of the diffusion equation for flat disc materials have been reported by Payne et al., (1986), Oliveira (2001), Lima (2002), Sablani and Raham, 2003, Rastogi and Raghavrao, (2004) and Carmo and Lima (2005).

The goals of modeling and simulation of water loss rate include improving and optimizing design as well as developing better insight into the working of the process ultimately leading to the optimal operation and control of the process. Attempts were made in this work to develop a mathematical model that describes rate of water loss during osmotic dehydration of red bell pepper at different field strengths osmotic solutions and process times.

2.0 MATHEMATICAL MODELLING

Mathematical model for water transfer rate during the process was developed, using a sheet of bell pepper tissue, from first principle approach: water diffusion into and out of the tissue (sliced red bell pepper with characteristic length of 0.0064mm). Conservation of water in differential element of the tissue was accomplished by identifying the simplifying assumptions, defining appropriate initial and boundary...
conditions. The following assumptions were made: Homogeneous tissue is assumed and one-dimensional diffusion occurs; the initial water concentration in tissue is uniform, diffusing water enters through the plane faces and negligible amount through the edge. Two simultaneous counter-current phenomena are assumed in modeling: water diffusion into the osmotic solution and the osmotic solute into the bell pepper tissue as depicted in Figure 1 at different field strengths (0, 0.5, 1.5 & 2.0 kv) as well as osmotic solutions (sucrose only and sucrose/salt solution) as depicted in Table 1.

Figure 1: species conservation in a differential volume (elemental tissue of the pepper)

Statement of species conservation, from Figure 1, is:

Time rate of change of water in the tissue = Influx of water into the tissue – Out flux of Water from the tissue

\[ \frac{\partial c}{\partial t} = \frac{\partial (j A_x)}{\partial t} \] (1)

Influx of water into the tissue = \( j \) (2)

Out flux of Water from the tissue = \( j + \frac{\partial j}{\partial x} A_x \) (3)

Substitute equation 1, 2, 3 into statement of species conservation and gives:

\[ \frac{\partial (j A_x)}{\partial t} = j - \left( j + \frac{\partial j}{\partial x} A_x \right) \] (4)

Divide equation (5) by \( A_x \) and gives:

\[ \frac{\partial c}{\partial t} = - \frac{\partial j}{\partial x} \] (6)

Recall, Fick’s first law:

\[ J = -D_{eff} \frac{\partial c}{\partial x} \] (7)

\[ D_{eff} = D \cdot \varepsilon \] (8)

\[ D_{eff} = \text{Effective diffusion coefficient} \]

\[ \varepsilon = \text{Open pore porosity of the tissue} \]

\[ \frac{\partial c}{\partial t} = \frac{\partial^2 c^*}{\partial \eta^2} \] (9)

Now, combine equation 6, 7 and 8 and obtain:

\[ \frac{\partial c}{\partial t} = \frac{\partial}{\partial x} \varepsilon D \frac{\partial c}{\partial x} \] (9)

Equation 9 is an appropriate equation predicting water transfer rate during osmotic dehydration of bell pepper tissue pre-treated by pulsed electric field.

The following initial and boundary conditions were used:

**Initial condition**:

\[ C(t = 0, x) = C_0 \] (10)

**Boundary conditions**:

\[ C(x = 0, t) = C_e \] (11)

\[ C(x = L, t) = C_e \] (12)

Defining the following dimensionless parameters:

Dimensionless space variable:

\[ \eta = \frac{x}{L} \] (13)

Dimensionless time variable:

\[ \tau = \frac{D_{eff} t}{L^2} \] (14)

Now, introduce dimensionless concentration function:

\[ C^* = \frac{c - c_e}{C_0 - c_e} \] (15)

Then, the dimensionless function changes the initial and boundary conditions to:

\[ C^*(\tau = 0, \eta) = 1 \] (16)

\[ C^*(\tau, 0) = 0 \] (17)

\[ C^*(\tau, 1) = 1 \] (18)

\[ \frac{\partial c^*}{\partial \tau} = \frac{\partial^2 c^*}{\partial \eta^2} \] (19)
Equation 19 is a dimensionless equation, obtained from equations 13 – 18, predicting rate of water transfer during the process.

2.1 ANALYTICAL PROCEDURE

Equation (19) is solved through separation of variables technique, which is, seeking a solution in which the time variable (τ) is separated from space variable (η) i.e.

$$C^*(\eta, \tau) = W(\tau)Y(\eta)$$  \hspace{1cm} (20)

- $W(\tau) = \eta$-independent function
- $Y(\eta) = \tau$-independent function

$$\frac{\partial^2 C^*}{\partial \tau^2} = \frac{d^2 Y(\eta)}{d\eta^2} W(\tau)$$  \hspace{1cm} (21)

Equation 21 represents L.H.S of equation 19

While equation 22 represents R.H.S of equation 19

Now, equate (21) and (22), thus equation 19 becomes:

$$\frac{dW(\tau)}{d\tau} Y(\eta) = \frac{d^2 Y(\eta)}{d\eta^2} W(\tau)$$  \hspace{1cm} (23)

Collect the like variables and equal to:

$$\frac{1}{W(\tau)} \frac{dW(\tau)}{d\tau} = \frac{1}{Y(\eta)} \frac{d^2 Y(\eta)}{d\eta^2}$$  \hspace{1cm} (24)

Then, let both sides of equation (24) equal to separation constant ($-\lambda^2$ )then, solve equation 24 independently.

$$\frac{1}{W(\tau)} \frac{dW(\tau)}{d\tau} = -\lambda^2$$  \hspace{1cm} (25)

$$\frac{dW(\tau)}{d\tau} + W(\tau)\lambda^2 = 0$$  \hspace{1cm} (26)

$$\frac{1}{Y(\eta)} \frac{d^2 Y(\eta)}{d\eta^2} = -\lambda^2$$  \hspace{1cm} (27)

$$\frac{d^2 Y(\eta)}{d\eta^2} + Y(\eta)\lambda^2 = 0$$  \hspace{1cm} (28)

Equations (26) & (28) are ordinary differential equations. Partial differential equation (19) has been reduced to two ordinary differential equations. Integrated form of equation (26) is:

$$\frac{dW(\tau)}{W(\tau)} = -d\tau \lambda^2$$

$$W = C e^{-\lambda^2 \tau}$$  \hspace{1cm} (29)

The general solution for equation 28 is:

$$Y(\eta) = A \cos(\lambda \eta) + B \sin(\lambda \eta)$$  \hspace{1cm} (30)

Necessary dimensionless boundary conditions (equations 16, 17 and 18) are applied and equation 31 is obtained:

$$Y(\eta) = B \sin(\lambda \eta)$$  \hspace{1cm} (31)

Since B=0, it yields a trivial solution, then, $B \neq 0$ we must have:

$$B \sin(\lambda \eta) = 0.$$  Since $\sin \pi = 0$$

Then, set $B = 1$

Thus, $B \sin \lambda = \sin \pi$, and divide both side by $\sin:

$$\lambda = \pi$$

Then, discrete values are assumed, the value must then, be of the form:

$$\lambda = n\pi$$

Therefore, $\lambda = n\pi$, for $n=1,2,3 \ldots$

Substituting the Eigenvalue to 31 to obtain

$$Y(\eta) = \sin(n\pi \eta)$$  \hspace{1cm} (32)

Hence, combine 29 and 32 to give non trivial solution of diffusion equation (20) which satisfies the two boundary conditions:

$$C^*_n(\tau, \eta) = B_n \sin(n\pi \eta)(C_n e^{-(n^2 \pi^2 \tau)})$$

Where $E_n = B_n C_n$ is arbitrarily constant.

Now, a series equation is formally formed:

$$C^*(\tau, \eta) = \sum_{n=0}^{\infty} E_n \sin(n\pi \eta)(e^{-(n^2 \pi^2 \tau)})$$  \hspace{1cm} (33)

Then, apply initial condition:

$$C^*_n(0, \eta) = \sum_{n=0}^{\infty} E_n \sin(n\pi \eta)(e^{-(n^2 \pi^2 \tau)})$$

But $(e^{-(n^2 \pi^2 \tau)}) = 1$

Recall dimensionless initial condition: $C^*_n(0, \eta) = 1$

Then,$C^*_n(0, \eta) = \sum_{n=0}^{\infty} E_n \sin(n\pi \eta)(1) = 1$

Now, orthogonal properties were applied, then, integrate the function across the domain so as to determine $E_n$:

$$\int_0^1 \sum_{n=0}^{\infty} E_n \sin(m\pi \eta) . \sin(n\pi \eta) = \int_0^1 \sin m \pi x$$  \hspace{1cm} (34)
The sum of LHS of (34) results in many terms, however they are zero except for the one when \( n=m \). Then, equation 34 becomes:

\[
\frac{1}{2} E_n = \int_0^1 \sin(n\pi\eta)\, d\eta
\]

RHS of this expression is integrated analytically:

\[
\int_0^1 \sin(n\pi\eta)\, d\eta = \frac{\cos\pi n - 1}{n\pi}
\]

RHS = \( \frac{1 - \cos\pi n}{n\pi} \)

Now, RHS = LHS

\[
E_n = \frac{1}{2} E_n = \frac{1 - \cos\pi n}{n\pi}
\]

Then, substitute dimensionless variables and equation 35 into equation 33 to obtain 35a, also make \( C_t \) as the subject of equation 33. Now, determine concentration of water in the tissue at different operating conditions.

\[
C_{WT} = C_{WE} + 2(C_{SO} - C_{SE}) \sum_{n=0}^{\infty} \left( \frac{1 - \cos\pi n}{n\pi} \right) (\sin(n\pi\eta)) \left( e^{-\frac{n^2\pi^2}{(L_D\epsilon)} \frac{Dt}{L^2}} \right)
\]

(35a)

Therefore, equation 35a predicts concentration of water in the tissue at any process time. Then, equation 36 estimates water loss \( \Delta M_w = \frac{M_w C_{wt} - M_o C_{wo}}{M_o} \) during the process:

\[
\text{Water loss} \ (\Delta M_w) = \frac{M_w C_{wt} - M_o C_{wo}}{M_o}
\]

(36)

\[ M_o = \text{Mass of the tissue at time (t) = 0} \]

\[ M_t = \text{Mass of the tissue at time (t) = t} \]

\[ C_t = \text{Concentration of water in the tissue at time (t) = t} \]

\[ C_0 = \text{Concentration of water in the tissue at time (t) = 0} \]

Table 1: Properties of flat sheet of the pepper used in the simulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>30</td>
</tr>
<tr>
<td>Characteristic length (L)</td>
<td>0.0064m</td>
</tr>
<tr>
<td>Initial solute concentration in the tissue for binary solution</td>
<td>0.07486</td>
</tr>
<tr>
<td>Initial water concentration in the tissue for ternary solution</td>
<td>0.89564</td>
</tr>
<tr>
<td>Initial water concentration in the tissue for binary solution</td>
<td>0.09328</td>
</tr>
<tr>
<td>Initial solute concentration in the tissue for ternary solution</td>
<td>0.91448</td>
</tr>
</tbody>
</table>

Source: Ade-Omowaye et al., 2002

Table 2: Pulsed electric field pretreatment conditions and diffusivities at 400 use pulse number

<table>
<thead>
<tr>
<th>Field strength (kv/cm)</th>
<th>Osmotic solution</th>
<th>Diffusivity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Untreated tissue</td>
<td>Ternary solution</td>
<td>2.32 \times 10^{-10}</td>
</tr>
<tr>
<td>1.0</td>
<td>Ternary solution</td>
<td>6.79 \times 10^{-10}</td>
</tr>
<tr>
<td>1.5</td>
<td>Ternary solution</td>
<td>7.01 \times 10^{-10}</td>
</tr>
<tr>
<td>2.0</td>
<td>Ternary solution</td>
<td>7.01 \times 10^{-10}</td>
</tr>
<tr>
<td>Untreated tissue</td>
<td>Binary solution</td>
<td>1.35 \times 10^{-10}</td>
</tr>
<tr>
<td>1.0</td>
<td>Binary solution</td>
<td>3.3 \times 10^{-10}</td>
</tr>
<tr>
<td>1.5</td>
<td>Binary solution</td>
<td>3.44 \times 10^{-10}</td>
</tr>
<tr>
<td>2.0</td>
<td>Binary solution</td>
<td>3.57 \times 10^{-10}</td>
</tr>
</tbody>
</table>

Source: Ade-Omowaye et al., 2002

2.2 COMPUTER SIMULATION

MATLAB was used as simulation tool to solve the corresponding equations (Equations 35a, and 36). It is a high-performance language for technical computing, which is characterized by its ability to perform mathematical calculations in this work and it has large library of built-in functions and graphical visualization tools. Therefore, resulting equations from the analytical approach were implemented in MATLAB by developing codes in it so as to obtain water loss during the process at different field strengths (1.0, 1.2, 1.5, 1.7, 2.0 Kv/cm) in different osmotic solutions as indicated in Table 2. Then, correlation
coefficient for analytical results with experimental data was carried out in standard program in excel Microsoft spreadsheet. The agreement between predicted and experimental results was further evaluated using the Mean Relative Deviation Modulus (MRDM) (%E) as indicated in equation 37.

\[
\%E = \frac{1}{n} \sum \left| \frac{V_E - V_P}{V_E} \right |
\]  

(37)

Where: \(V_E\) = Experimental value  
\(V_P\) = Predicted value

3. RESULTS AND DISCUSSION

Figures 2 to 9 show plots of the experimental and predicted loss rate during osmotic dehydration of red bell pepper at different field strengths (1, 1.2, 1.5, 1.7 and 2.0 kv/cm) in two osmotic solutions. A close relationship between the experimental and predicted water loss was observed. The value for mean relative deviation was used to measure the reliability of prediction. According to Azoubel and Murr (2002), value of MRDM less than or equal to 10% indicates good prediction of experimental data and the lower the percentage the better the model for predictive purpose. This indicates that the model for water transfer during the process predicts satisfactorily. The curves followed an exponential trend in all the figures for rate of water loss. This agrees with the earlier on the use of (Pulsed Electric Field) PEF as a pretreatment process for osmotic dehydration (Rastogi et al., 1999). The coefficient of determination between the predicted and experimental data was very high and ranged from 0.955 to 0.997. The correlation coefficients are near one which indicates positive correlation. The predicted results are similar to earlier work by Farias (2002). The predicted results in Figure 1 for the untreated, using sucrose solution only, revealed about 37% of water was lost during the process. However, with the application of different field strengths using the same osmotic solution (sucrose only), the predicted results, in Figure 2 and Figure 4, revealed that rate of water loss was improved with 23-24%. The trend of predicted water loss is similar to earlier work on kinetics of osmotic dehydration of red bell pepper as influenced by PEF (Ade-Omowaye et al., 2002). It was also noted from the predicted results that there was no appreciable difference in water loss values using field strength of 1.5 and 2.0kv/cm as shown in Figure 3 and 4.

With second osmotic solution (sucrose/salt solution) the predicted water loss value, as indicated in Figure 5 to Figure 8, for untreated tissue it was about 28% water loss, but there was an improvement of 7%-9% in the amount of amount of water released by (Pulsed Electric Field) PEF-pretreated tissue compared to the untreated one. Thus, in both solutions, it was predicted that with the application of different field strengths resulted in an increase of 9-23% as compared with the untreated tissue. It was also observed, from all the Figures for water loss that the rate of water loss was faster at the beginning of the process compared to amount released between 10,000sec. and 25,000sec. The predicted water loss values suggest that the rate of water transfer was highest at the beginning of the dehydration. This might be due to large gradient of the osmotic pressure between osmotic solution and the tissue (Salvatori et al., 1998).
4. CONCLUSION

Analytical method of solution of diffusion equation was developed and applied to water transfer rate during osmotic dehydration in a flat sheet of red bell pepper tissue. The following conclusions were made based on the analysis of the results obtained at different operating conditions: Satisfactory prediction, with less than 10% of mean relative deviation modulus of rate of water transfer at different operating conditions was obtained. Better rate of water transfer was obtained for predicted value of pretreated tissue compared to the untreated tissue. It could be concluded that combined PEF and osmotic dehydration would yield satisfactory water loss during the process. Therefore, modeling of water loss could be used for optimizing design and control of the process.
REFERENCES


Park, K. J; Bin, A; and Brod F. P. R. (2002). Drying of Pears with and without Osmotic Dehydration: Journal of Food Engineering, 56, 97 – 103.


